Blind Multi-Stage Scoring Auctions with Two-Sided Incomplete Information for Government Procurement

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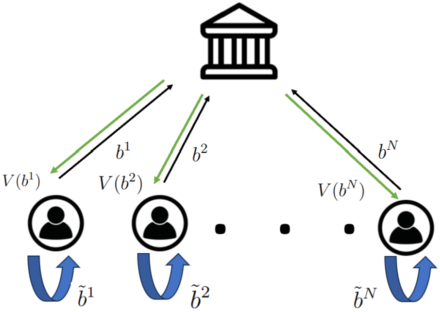
Setting



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- For $t = 1, \ldots, T$
  - N bidders submit bids b^i
  - Bidders receive score $V(b^i)$
  - bidders adjust bid $\tilde{b}^i$ and resubmit

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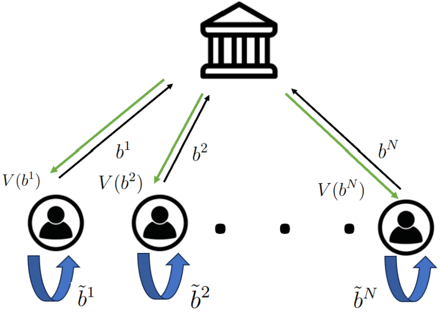

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How can bidders adjust their bids efficiently?

Value Functions

Auctioneer's Multi-Attribute Value Function

$$V(x) = \sum_{i=1}^{d} w_i \cdot v_i(x_i)$$

$$x \in \mathbb{R}^d, w \in \Delta_d = \left\{ \sum_{i=1}^{d} w_i = 1, w \geq 0 \right\}$$

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Single Attribute Value Functions

$$v_i(y) = \alpha_i^0 y + \alpha_i^1, \quad v_i(y) = \alpha_i^0 e^{\alpha_i^1 y}$$

$$v_i(y) = \begin{cases} \alpha_i^0 y + \alpha_i^1 & y \leq \alpha_i^5 \\ \alpha_i^3 y + \alpha_i^4 & \text{o.w} \end{cases}$$

Approximate Value Functions

Approximate Value Function

$$\tilde{V}(x) = \sum_{i=1}^{d} w_i \sum_{k=1}^{M} z_k^i \phi_k(x_i; \alpha_k^i)$$

$$\text{s.t} \quad z_k^i \in \{0, 1\}, \quad \sum_{k=1}^{M} z_k^i = 1 \quad \forall i \in \{1, \dots, d\}, \quad w \in \Delta_d$$

Library of Prototypes: $\mathcal{F} = \{\phi_k(y; \alpha_k)\}_{k=1}^{M}$

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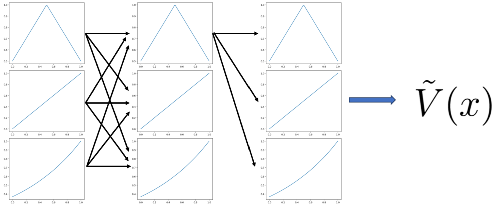

Value Function Estimation

Estimation Problem

After t rounds a bidder must solve

$$\min_{\{z_k^i,\alpha_k^i\}_{i,k=1}^{d,M},\{w_i\}_{i=1}^{d}} \frac{1}{t}\left[\sum_{(x^s,V(x^s))\in\mathcal{M}} (V(x^s) - \tilde{V}(x^s))^2\right]$$

$$\text{s.t} \quad z_k^i \in \{0,1\}, \quad \sum_{k=1}^{M} z_k^i = 1 \quad \forall i \in \{1,\ldots,d\}, \quad w \in \Delta_d$$

$$\mathcal{M} = \{(x^i, V(x^i)) : i = 1,\ldots,t\}$$

Online Greedy Solver

- Given a Value Function $V(x)$ and a Library of Prototypes $\mathcal{F}$
- Set $w^1 = [\frac{1}{d}, \ldots, \frac{1}{d}] \in \mathbb{R}^d$
- Assume 1 (Bid, Value) pair has been retrieved $\mathcal{M}^1 = \{(x^1, V(x^1))\}$
- For $t = 1, \ldots, T$
  - Solve

$$\phi_{k_i^*}(y; \alpha_k^{i*}) = \arg \min_{\phi_k \in \mathcal{F}} \min_{\alpha_k^i} \sum_{(x^s, V(x^s)) \in \mathcal{M}^t} (V(x^s) - w_i^t \phi_k(x_i^s; \alpha_k^i))^2 \quad \forall i \leq d$$

  - Solve

$$w^* = \arg \min_{w \in \Delta_d} \sum_{(x^s, V(x^s)) \in \mathcal{M}^t} \left(V(x^s) - \sum_{i=1}^{d} w_i \phi_{k_i^*}(x_i^s; \alpha_k^{i*}) \right)^2$$

  - $w^{t+1} \leftarrow w^*$
  - $\mathcal{M}^{t+1} \leftarrow \mathcal{M}^t \cup \{(x^{t+1}, V(x^{t+1}))\}$

Government Construction Bidding

- The Federal Acquisition Regulation (FAR) describes a best-value auction where agencies often assign scores across multiple criteria and combine them to form a total score
  - The criteria may include, Technical Merit, Creativity, and Personnel Experience
- Best Value Auctions are commonly used in procuring **Construction Contracts**

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Bidding Simulation

1. Elicit a value function $V(x)$
2. Apply greedy solver with a next bid selection rule of

$$x^{t+1} \leftarrow \arg\max_{x \in \mathbb{R}^d} \tilde{V}(x)$$

Step 1: Expert Elicitation

Attribute	Metric
Safety Score	EMR, OSHA incident rate
Personnel Experience	Avg. years of experience, certifications
Number of Similar Projects	Projects completed of comparable size and complexity
On-Time Completion History	Average completion time (% over or under planned time)
Project Manager Procedure	Quality of PM plan, tools used
Cost of Project Bid	Total bid estimate
Projected Duration	% over or under requested completion time

Table: Construction Bid Evaluation: Metrics for Each Attribute

Step 1: Expert Elicitation

Attribute	Swing Rank	Swing Rating	Swing Weight
Cost	1	100	0.1812
Number of Similar Projects	2	90	0.1630
Personnel Experience	3	87	0.1576
Safety Score	4	85	0.1540
Projected Duration	5	70	0.1268
Project Manager Procedure	6	65	0.1178
On-Time Completion History	7	55	0.0996

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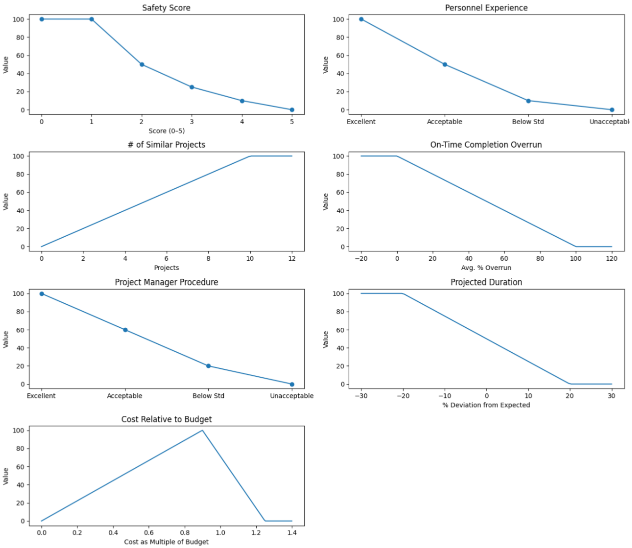
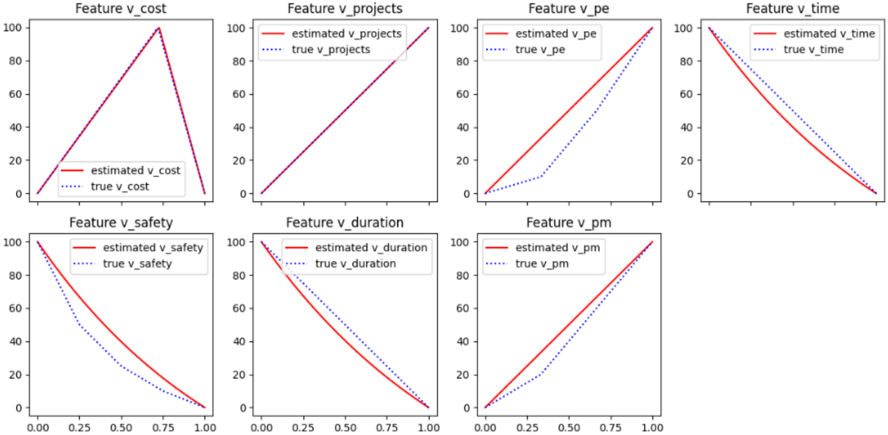


Step 2: Bidding Simulation ($T = 20$)



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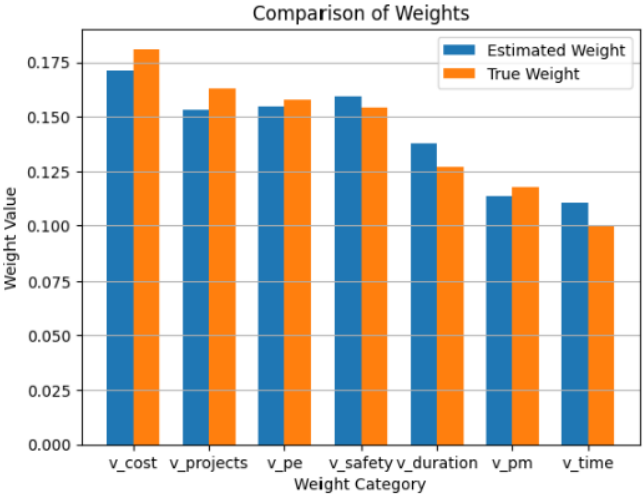

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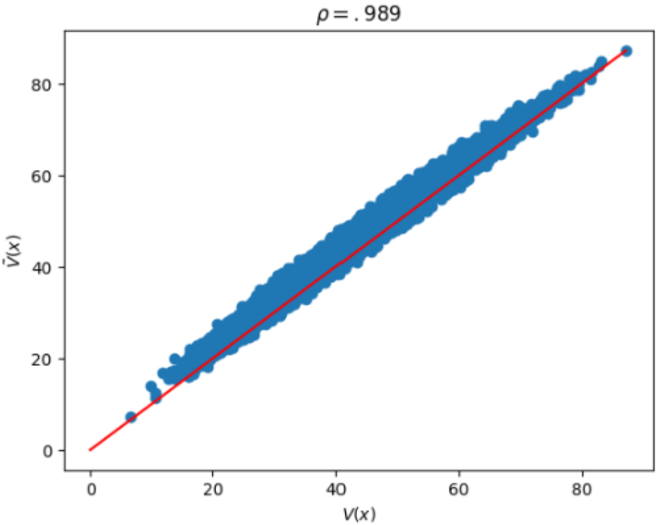

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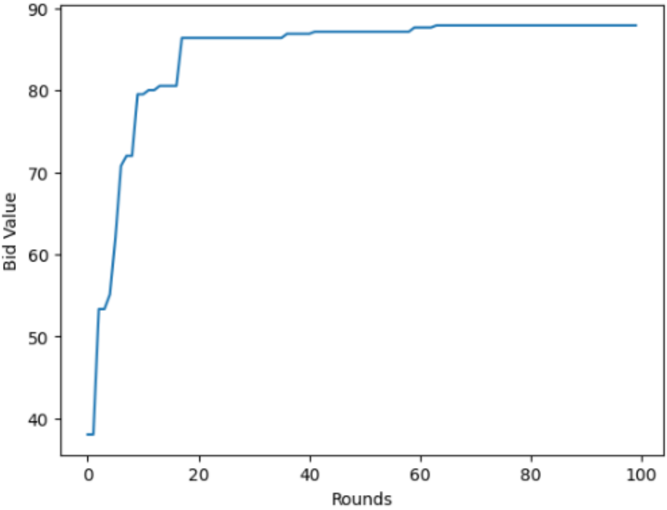

Known ϕ_k Case

Reduction with Known Structure

Given $\mathcal{H}^t = \{(x^s, V(x^s)) : s = 1, ..., t\}$ and $v_i = \phi_{k_i}, \quad i = 1, \ldots, d$

$$\min_{w \in \Delta_d} \frac{1}{2} w^T Z^T Z w - w^T Z v$$

$$Z = [\phi_{k_i}(x_i^s; \alpha_k^i)]_{s,i=1}^{t,d}, \quad v = V(x^s)$$

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